

Benchmark Validação Técnica

A seguinte seleção de 52 séries de dados de e projetos da análise incluídos neste Benchmark de validação representa:

Testes padrão da exatidão numérica para operações matemáticas do ponto flutuante (tais como o teste da variação, etc. relativos pequenos.);

Séries de dados de publicadas de benchmarks desenvolvidos com a finalidade de testar programas estatísticos, e usadas em revisões publicadas de estatística e dos pacotes de matemática (incluindo todos as séries de dados de benchmark propostos em "Benchmark Data Sets for Evaluating Microcomputer Statistical Programs," by Elliott, Reisch, and Campbell); e uma detalhada seleção de complexas amostras de série de dados e demanda de problemas numéricos, (algumas séries de dados incomuns), que nos foram recomendados por peritos em suas respectivas áreas da estatística e/ou publicados em livros de estatística e monografias especiais, incluindo representativas amostras de problemas computacionais, provenientes de "Analysis of Messy Data", por Milliken e Johnson (1984), e "Applied Linear Statistical Models", por Neter, Wasserman, e Kutner (1985), bem como os livros de Box et al., Cox, Lindman, Searle, e muitos outros autores.

Para sua melhor compreensão, **STATISTICA** é o único pacote estatístico disponível no mercado que foi bem sucedido, passando em todos os testes incluídos neste set de benchmarks (e alguns testes descritos aqui não passaram em nenhum outro programa, a não ser no **STATISTICA**).

Example 1: The "small relative variance test" of numerical precision In the following sample data set, variable var2 (the second column) which features a small relative variance is a linear function of var3 (the third column); thus, the correlation coefficient between any variable (e.g., variable var1) and var2 should be identical to the correlation between that variable and var3.

var1	var2	var3
1.0	100000.00000001	1.0
2.0	100000.00000002	2.0
3.0	100000.00000001	1.0
4.0	100000.00000002	2.0
5.0	100000.00000001	1.0
6.0	100000.00000002	2.0
7.0	100000.00000005	5.0

Here are the two correlation coefficients (var1*var2 and var1*var3) calculated by **STATISTICA**(using its extended precision optimization algorithm), and displayed with the highest precision available.

variables	Pearson r	p-level
var1 * var2	0.65465367070798	0.111
var1 * var3	0.65465367070798	0.111

To our knowledge, **STATISTICA** is the only program available on the market that will correctly compute these correlations (or correlations from other data sets featuring very small relative variances).

Example 2: A medium size multi-factor unbalanced ANOVA design

The following design is a 5 x 5 x 5 x 3 (between-group) x 3 x 3 x 3 (repeated measures) design (with unequal N). Thus there are 375 groups and 27 dependent variables (data file ANOVA4 is available from StatSoft). The between-group design matrix for the highest order interaction has 128 degrees of freedom. Shown below are the univariate and multivariate results for the highest order (7-way) interaction.

general manova	INTERACTION: 1 x 2 x 3 x 4 x 5 x 6 x 7 1-IV1, 2-IV2, 3-IV3, 4-IV4, 5-RFACT1, 6-RFACT2, 7-RFACT3				
Univar. Test	Sum of Squares	df	Mean Square	F	p-level
Effect	8664.99	1024	8.461903	1.02411	31744
Error	24854.14	3008	8.262680		

Test	Value	p-level
Wilks' Lambda	.088651	.29812
Rao R (1024,2966)	1.027036	
Pillai-Bartlett Trace	2.071145	.30355
V(1024.3008)	1.026166	

Example 3: A medium size multi-factor unbalanced ANOVA design (very large and very small values)

Example 3.1. In the first part of this test, the data set used in the previous example (Example 2, original range of values 0.1 to 10.0) was transformed by multiplying each dependent variable in the original data set by 100,000; then, the analysis of variance reported in the previous example was performed on the transformed data. Shown below are the univariate and multivariate results for the highest order (7-way) interaction (cf. Example 2).

Univar. Test	Sum of Squares	df	Mean Square	F	p-level
Effect	8664.99	1024	8.461903	1.02411	31744
Error	24854.14	3008	8.262680		

Test	Value	p-level
Wilks' Lambda	.088651	.29812
Rao R (1024,2966)	1.027036	
Pillai-Bartlett Trace	2.071145	.30355
V(1024.3008)	1.026166	

Example 3.2. In the second part of this test, the data set used in Example 2 (original range of values 0.1 to 10.0) was transformed by dividing each dependent variable in the original set by 100,000; the analysis of variance reported in Example 2 was then performed on the transformed data. Shown below are the univariate and multivariate results for the highest order (7-way) interaction (cf. the first part of this example and Example 2).

Univar. Test	Sum of Squares	df	Mean Square	F	p-level
Effect	8664.99	1024	8.461903	1.02411	31744
Error	24854.14	3008	8.262680		

Test	Value	p-level
Wilks' Lambda	.088651	.29812
Rao R (1024,2966)	1.027036	
Pillai-Bartlett Trace	2.071145	.30355
V(1024.3008)	1.026166	

Example 4: A large multi-factor unbalanced ANOVA design

The following design is a 20 x 10 x 2 x 2 (between-group) x 3 (repeated measures) design with unequal N. Thus, there are 800 groups and 3 dependent variables (data file ANOVA44 is available from StatSoft). The between-group design matrix for the highest order interaction has 171 degrees of freedom. Shown below are the univariate and multivariate results for the highest order (5-way) interaction.

general manova					
INTERACTION: 1 x 2 x 3 x 4 x 5					
1-COUNTRY, 2-RAINFALL,3-REGION, 4-STATUS, 5-RFACTOR					
Univar. Test	Sum of Squares	df	Mean Square	F	p-level
Effect	17.9462	342	.052474	.92406	.82876
Error	181.8289	3202	.056786		

general manova		
INTERACTION: 1 x 2 x 3 x 4 x 5		
1-COUNTRY, 2-RAINFALL, 3-REGION, 4-STATUS, 5-RFACTOR		
Test	Value	p-level
Wilks' Lambda	.826507	.78876
Rao R (342,inf)	.935296	
Pillai-Bartlett Trace V(342,3202)	.181690 .935531	.78788

To our knowledge, *STATISTICA* is the only program available on the market that can process ANOVA designs of this size.

Example 5: Precision of ANOVA routines (small within-cell variances relative to the between-group variance)

Here is a test of the precision of computations in ANOVA: A data file was created with 10 cases and 5 groups (2 cases per group), and 12 dependent variables. The groups in the grouping variable *IV* were coded 1 through 5. The dependent variables *DV_i* (*i* = 1 to 12) were then computed as $DV_i = IV \text{ casenumber} / 10^{**i}$ (each successive dependent variable was computed as a constant plus the case number divided by 10 to the power of *i*). This results in small within-cell variances relative to the between-group variance.

general manova			
MAIN EFFECT: IV			
1-IV			
depend. variable	Mean Sqr Effect	Mean Sqr Error	F(df1,2) 4,5
DV1	5.202000	.00005	104040.
DV2	5.020020	.0000005	1004E4
DV3	5.002000	.5E-8	10004E5
DV4	5.000200	.5E-10	100004E6
DV5	5.000020	.5E-12	1E13
DV6	5.000002	.5E-14	1E15
DV7	5.000000	.5E-16	1E17
DV8	5.000000	.5E-18	1E19
DV9	5.000000	.5E-20	1E21
DV10	5.000000	.500E-22	99996E18
DV11	5.000000	.500E-24	99996E20
DV12	5.000000	.502E-26	99584E22

To our knowledge, *STATISTICA* is the only program available on the market that will correctly compute the within MS error component for all dependent variables in this design.

Examples 6 and 7: Logistic regression, maximum likelihood

Example 6. Cox (1970, p. 86) reports data describing the failure (variable *Failure*) of objects as a function of time (*Time*). Cox fitted the data by the logistic model. Shown below are the maximum likelihood estimates and their standard errors produced via *STATISTICA*: Nonlinear Estimation (see also Brown et al., 1983, p. 317).

nonlin. estimat.		
Parameter Estimates		
Std. Errs were computed after scaling MS-err. to 1.		
Param.	Const.	TIME
Estimate	-5.415	.0807
Std. Err.	.728	.0224
t(5)	-7.438	3.6099
p-level	.00069	.0154

Example 7. A data set reported in Neter, Wasserman, and Kutner (1985, p. 365) describes the results of a study of coupon redemption. The coupons differed in their value, that is, with regard to the price reduction offered. The dependent variable of interest is how many coupons of

each type were redeemed. Shown below are the maximum likelihood parameter estimates for the logistic regression model computed by *STATISTICA*: Nonlinear Estimation (weighted least squares estimates are reported in Neter et al., p. 365).

nonlin. estimat. Parameter Estimates		
Std. Errs were computed after scaling MS-err. to 1.		
Param.	Const.	REDUCTN
Estimate	-2.185	.1087
Std. Err.	.165	.0089
t(8)	-13.267	12.2894
p-level	.000	.0000

Example 8: Exponential regression, ordinary least squares

This example is based on a data set reported in Neter, Wasserman, and Kutner (1985, p. 469). The data contain information on the number of days that each of 15 severely injured patients were hospitalized (variable *Days*) and an index of the prognosis for long-term recovery for each patient (variable *Prognos*). Shown below are the parameter estimates produced by *STATISTICA*: Nonlinear Estimation for the exponential regression model: $Prognos = g_0 * \exp(g_1 * Days)$ [g_0 and g_1 are parameters], the loss function is least squares (see also Neter et al., p. 478, Table 14.3).

nonlin. estimat. Parameter Estimates		
Param.	g_0	g_1
Estimate	58.60662	-.0396
Std.Err.	1.54984	.0019
t(13)	37.81474	-20.8667
p-level	.00000	.0000

Example 9: User-defined (exponential) regression, ordinary least squares

The data set for this example is again based on Neter, Wasserman, & Kutner (1985, p. 484). To study the efficiency of two new manufacturing plants, a ratio was computed of the per-unit-production cost expected in a modern facility after learning has occurred, over the actual per-unit-production cost for selected weeks over a 90-week span. Neter et al. fit the following model to these data: $y = b_0 b_1 * x^{g_2} b_3 * \exp(b_2 * x)$, where xg is an indicator variable to denote the two plants, x denotes the number of weeks, y is the efficiency index, and $b_0, b_1, b_2,$ and b_3 are parameters. This formula can be typed "as is" into the user-defined model specification editor. Shown below are the results computed by *STATISTICA*: Nonlinear Estimation (using the Rosenbrock pattern search method to find start values, followed by quasi-Newton iterations; Neter et. al. report the results on p. 484-485).

nonlin. estimat. Parameter Estimates				
$y = b_0 b_1 * x^{g_2} b_3 * \exp(b_2 * x)$				
Param.	B_0	B_1	B_3	B_2
Estimate	1.0156	-.0473	-.5524	-.1348
Std. Err.	.0037	.0041	.0083	.0046
t(26)	274.5491	-11.5026	-66.6689	-29.5186
p-level	0.0000	.0000	.0000	.0000

Example 10: Discontinuity (breakpoint) in regression function

This example is also based on a data set reported in Neter, Wasserman, & Kutner (1985, p. 348). Specifically, the data set pertains to a production process in which the per-unit cost is related to the lot size. Supposedly, for lots greater than 500, the relationship between the variables changes; Neter et al. (1985) fit a linear model that allowed for different slopes for lots of sizes less than or equal to 500, and lots greater than 500. Specifically, Neter et al. fit the following model: $y = b_0 b_1 * x^{b_2} * (x > 500)$ ($b_0, b_1,$ and b_2 are parameters). In this model, the logical expression $(x > 500)$ serves as a multiplier: If the expression is true, it will evaluate to 1, if it is false, it will evaluate to 0. Therefore, this equation actually represents two models: $y = b_0 b_1 * x$ for $x \leq 500$, and $y = b_0 b_1 * x^{b_2} * (x - 500)$ for $x > 500$. The model can again be typed in to the user-model specification editor "as is"; shown below are the parameter estimates computed by *STATISTICA*: Nonlinear estimation (see Neter et al., p. 348).

Parameter Estimates			
$y = b_0 b_1 * x^{b_2} * (x > 500)$			
Param.	B_0	B_1	B_2
Estimate	5.895447	-.00395	-.00389
Std. Err.	.604213	.00149	.00231
t(5)	9.757232	-2.64990	-1.68515
p-level	.000192	.04543	.15277

Example 11: Weighted Least Squares

Weighted least squares or any other (user-specified) loss function can be specified in *STATISTICA*: Nonlinear Estimation. An example of weighted least squares is presented in Neter et al. (1985, p. 169). The example data set contains information concerning the cost for preparing a bid and the size of the bid. Neter et al. fit a linear regression model ($Bid\ cost = b_0 + b_1 * Bid\ size$), using the residuals weighted by the inverse of the squared *Bid size* values in the loss function [$Loss = ((Predicted - Observed) ** 2) * (1 / Bid\ size ** 2)$]. Here are the results computed by *STATISTICA*: Nonlinear Estimation (see Neter et al., p. 169-170).

Parameter Estimates $y = b_0 * x$		
Param.	B0	B1
Estimate	5.656852	4.19055
Std.Err.	.965238	.40366
t(10)	5.860577	10.38127
p-level	.000159	.00000

Example 12: Robustness against collinearity problems (a linear model test of accuracy of nonlinear estimation)

The so-called *Longley* data (Longley, 1967) is a well-known data set for testing linear-least-squares regression programs for their ability to handle regression problems with redundant predictor variables (this data set is also referenced below for *STATISTICA*: Multiple Regression, Example 27). In this example, it will be used to test the accuracy of the general nonlinear estimation module of *STATISTICA*. In the user-model specification editor of *STATISTICA*: Nonlinear Estimation, we can specify the linear regression model, and request least squares parameter estimates. The parameter estimates computed by *STATISTICA*: Nonlinear Estimation (via *quasi-Newton* iterations) and their (asymptotic) standard errors (computed via finite difference approximation) are shown below (for comparison, see also Elliott, Reisch, & Campbell, 1989, p. 296). Note that *STATISTICA*: Multiple Regression will reproduce the parameter estimates with all 12 digits of precision.

multiple regress. Parameter Estimates				
Param.	A	B1	B2	B3
Estimate	-34822E2	15.06195	-.03582	-2.02023
Std.Err.	890420.	84.91493	.03349	.48840

multiple regres. Parameter Estimates			
Param.	B4	B5	B6
Estimate	-1.03323	-.051103	1829.155
Std.Err.	.21427	.226073	455.479

Example 13: Unbalanced ANOVA designs (Type I and III Sums of Squares)

Milliken and Johnson (1984, p. 129) discuss in some detail the analysis of a 2 x 3 unbalanced (due to unequal N) between-group design. Shown below are the summary ANOVA tables for that design; both the results for Type I Sequential Sums of Squares (see Milliken & Johnson, p. 142) and Type III Sums of Squares (see Milliken & Johnson, p. 132) are shown below.

general manova Summary of all Effects (Type I SS); design: 1-T, 2-B						
Effect	df Effect	MS Effect	df Error	MS Error	F	p
*1						
*2	1	76.563	10	2.0000	38.281	.0001
*12	2	45.372	10	2.0000	22.686	.0002
	2	35.815	10	2.000	17.908	.0005

general manova Summary of all Effects (Type I SS); design: 1-T, 2-B						
Effect	df Effect	MS Effect	df Error	MS Error	F	p
*1						
*2	1	61.714	10	2.0000	30.857	.0002
*12	2	38.585	10	2.0000	19.292	.0004
	2	35.815	10	2.000	17.908	.0005

Example 14: A 2-way nested design

Lindman (1974, p. 167) discusses a two-way nested design where factor *A* has three levels, and factor *B* has six levels, with two levels each nested in each level of factor *A*. Here is the results summary computed by *STATISTICA*: ANOVA/MANOVA (see Lindman, p. 172).

general manova						
Summary of all Effects; design: 1-A, 2-B						
Effect	df Effect	MS Effect	df Error	MS Error	F	p
*1	2	114.67	12	4.8889	23.455	.0001
*2	3	46.83	12	4.8889	9.580	.0017

Example 15: A 3-way nested design with customized error term

Milliken & Johnson (1984, p. 418) present an example of a 3-way nested design. In this experiment, male and female subjects were randomly assigned to one of 9 environmental chambers; the 9 environmental chambers, in turn, were assigned to 3 levels of a temperature factor. Thus, in this design *Chamber* is nested in *Temperature*, and subjects are nested in chambers. To produce the table of sums of squares as presented in Milliken & Johnson (1984, p. 419), the *Gender* by *Chambers* interaction was pooled into the error term before computing the table of all effects.

general manova		
Summary of all Effects; design: 1-TEMPERAT, 2-GENDER, 3-CHAMBER Customized Error Term		
Effect	df Effect	MS Effect
1	2	79.194
2	1	3.361
3	6	11.083
12	2	7.861
Error	24	1.653

Example 16: A nested design with a random effect

STATISTICA: ANOVA/MANOVA will automatically handle random effects. Lindman (1974, p. 173) shows an example of a nested design, where the nested factor is random. Factor *A* has four levels, factor *B* has 3 levels, and factor *C*(subjects) is a random effect with 9 levels. Shown below is the summary table for this design (see Lindman, 1974, p. 178).

general manova						
Summary of all Effects; design: 1-A, 2-B,3-C						
Effect	df Effect	MS Effect	df Error	MS Error	F	p
*1	3	86.44	18	8.53	10.14	.0004
*2	2	365.08	6	44.31	8.24	.0190
3	6	44.31	18	8.53	1.31	.3016
12	6	11.19				
13	18	8.53				

Example 17: Weighted means analysis of a nested design with unequal N (and missing cells)

The next example was taken from Searle (1987, p. 62). The data presented there describe a two-way nested classification of student opinions concerning computers. There were two classes -- *English* and *Geology* (factor *Course*) -- with different numbers of sections (taught by different teachers): *English* had two sections, *Geology* had 3 sections. To test the main effect for *Course*, Searle constructs a weighted means comparison. Shown below is the result of that comparison as computed by *STATISTICA*: ANOVA/MANOVA (see also Searle, 1987, p. 71).

general manova					
Planned Comparison 1-COURSE,2-SECTION					
Univar. Test	Sum of Squares	df	Mean Square	F	p
Effect	24.0000	1	24.0000	6.462	.0386
Error	26.0000	7	3.7143		

Example 18: A split-plot design with customized error term

Milliken and Johnson (1984, p. 297) present an example of a split-plot design. The design pertains to the effectiveness of 4 different fertility regimes on two varieties of wheat. Each of the four fertilizer levels was randomly assigned to one whole plot within each of one of two blocks. Shown below are the results for the *Fertility* and the *Variety* factor with the appropriate error terms (see Milliken & Johnson, 1984, p. 299).

general manova					
MAIN EFFECT: FERTILTY ERROR: FERTILTY x BLOCK					
Univar. Test	Sum of Squares	df	Mean Squares	F	p
Effect	40.1900	3	13.3967	5.802	.0914
Error	6.9275	3	2.3092		

general manova					
MAIN EFFECT: VARIETY ERROR: VARIETY x BLOCK FERTILTY x VARIETY x BLOCK					
Univar. Test	Sum of Squares	df	Mean Squares	F	p
Effect	2.25000	1	2.2500	1.068	.3599
Error	8.43000	4	2.1075		

Example 19: Strip-plot designs

Milliken and Johnson (1984, p. 320) discuss an experiment on the relationship between two irrigation methods and three levels of nitrogen on the yield of wheat. Again, the analysis requires the specification of custom error terms. All sums of squares are automatically computed by *STATISTICA*: ANOVA/MANOVA for the *Table of all Effects*. Note that there is a typographical error in the table presented in Milliken and Johnson (p. 320); specifically the sum of squares for factor *Irrigation* is 570.4 (and not 507.4).

manova		
Summary of all Effects; design: 1-REPLICAT, 2-IRRIGAT, 3-NITROGEN		
Effect	df Effect	MS Effect
1	3	41.154
2	1	570.375
3	2	169.542
12	3	10.931
13	6	2.819
23	2	47.375
123	6	1.431

Example 20: Split-plot designs with unequal numbers of subplots

Milliken and Johnson (1984, p. 385) discuss an example of a such a design. Five patients suffering from depression were randomly assigned to one of two treatment conditions (*Treatment: Placebo vs. Drug*). They were then examined after one week and after five weeks; the dependent variable was the patients' depression score during those examinations. Two patients did not return for the second examination, creating an unequal number of subplots in the design. In *STATISTICA*: ANOVA/MANOVA the results were produced via analysis of covariance, with covariates coding the effect for subjects within-treatment conditions. Here are the Type III sums of squares for the effects of interest (for a discussion of the choice of error terms see Milliken & Johnson, p. 394).

manova		
Summary of all Effects; design: 1-TREATMNT, 2-WEEK		
Effect	df Effect	MS Effect
1	1	15.5648
2	1	24.0833
12	1	4.0833

Example 21: Youden square designs

An example of a 4 x 4 Youden square with three factors *A*, *B*, and *C* is presented in Lindman (1974, p. 209). Factor *A* is "rotated" in its position with respect to factor *B*. Here is the table of all effects computed by *STATISTICA*: ANOVA/MANOVA, with the *B x C* interaction as the error term (see also the results table in Lindman, page 209).

Effect	df	MS	df	MS	F	p
	Effect	Effect	Error	Error		
1	3	47.000	3	7.000	6.7143	.076
2	2	39.000	3	7.000	5.5714	.098

Example 22: A 4 x 11 nested design with unequal numbers of levels (missing cells)

Milliken and Johnson (1984, p. 415) present an example data set, comparing 11 insecticides produced by four different companies. One company makes three insecticides, another makes four, and the remainder make two each. The effect for *Insecticide* (nested within *Company*) was tested via planned comparisons. Shown below are the results computed by *STATISTICA*: ANOVA/MANOVA (note that these results are slightly different than those reported in Milliken and Johnson on page 422; the analysis reported there is not consistent, and a typographical error must have found its way into the presentation; e.g., compare the mean reported on page 417 for the last group with the data from page 415).

Univar. Test	Sum of Squares	df	Mean Square	F	p
	Effect	1500.58	7	214.369	3.743
Error	1260.00	2	57.273		

Example 23: A complex design with many missing cells (testing Type IV hypotheses)

Milliken and Johnson (1984, p. 202) discuss a complex example of a design with many missing cells. The design contains 3 factors: *Group* (2 levels; whether or not subject received food stamps), *Age* (classified into three groups), and *Race* (*black*, *hispanic*, *white*). For brevity, only the results for the main effect for *Race*, and for the *Race by Group* interaction are shown below (see also results reported for the so-called Type IV analysis in Milliken and Johnson, Table 17.2, p. 203).

Univar. Test	Sum of Squares	df	Mean Square	F	p
	Effect	11.68	2	5.8385	1.991
Error	2627.47	92	28.5595		

Univar. Test	Sum of Squares	df	Mean Square	F	p
	Effect	113.70	2	56.8517	1.991
Error	2627.47	92	28.5595		

Example 24: A 2 (between) x 3 x 3 (repeated measures) design with missing cells

This example is based on a (fictitious) data set reported in Winer (1962, p. 324). The design has two repeated measures factors, each with 3 levels. Shown below is the summary univariate ANOVA table as computed by *STATISTICA*: ANOVA/MANOVA (see also Winer, p. 328); the multivariate tests for the *Noise x Time* interaction are also shown.

Summary of all Effects; design: 1-NOISE, 2-TIME, 3-DIALS						
manova	df	MS	df	MS	F	p
Effect	Effect	Effect	Error	Error		
1	1		4	622.78	.75	.435
*2	2	468.17	8	29.36	63.39	.000
*3	2	1185.17	8	13.19	89.82	.000
*12	2	166.50	8	29.36	5.67	.029
13	4	25.17	8	13.19	1.91	.210
23	4	2.67	16	7.94	.34	.850
123	4	2.83	16	7.94	.36	.836

INTERACTION: 1 x 2 1-NOISE, 2-TIME, 3-DIALS		
manova	Value	p-level
Test		
Wilks' Lambda	.15607	
Rao R Form 2 (2,3)	8.11102	.06166
Pillai-Bartlett Trace	.84393	
V (2,3)	8.11102	.06166

Example 25: A multivariate repeated measures split-plot design

This example is based on data reported in the documentation for *Ganova* (Brecht & Woodward, 1985). The design is a multivariate repeated measures split-plot design with two between-group factors (2x3), two repeated measures factors (2x3), and two dependent variables. Shown below are the summary results computed by *STATISTICA*: ANOVA/MANOVA.

Summary of all Effects; design: 1-A, 2-B, 3-FACTOR3, 4-FACTOR4					
general manova	Wilks' Lambda	Rao's R	df 1	df 2	p
Effect					
1	.75263	.8217	2	5	.4914
2	.71467	.4573	4	10	.7656
3	.62400	1.5064	2	5	.3076
4	.66764	.3734	4	3	.8175
12	.28476	2.1849	4	10	.1442
13	.52986	2.2183	2	5	.2044
23	.40968	1.4059	4	10	.3008
14	.54561	.6246	4	3	.6777
24	.20559	.9041	8	6	.5654
34	.81136	.1744	4	3	.9376
123	.23354	2.6732	4	10	.0945
124	.24201	.7746	8	6	.6410
*134	.06450	10.8778	4	3	.0394
234	.13046	1.3265	8	6	.3756
1234	.02388	4.1035	8	6	.0512

Example 26: Multivariate analysis of covariance, multivariate tests of parallelism

In this example we will specify a multivariate analysis of variance design with multiple covariates and test the parallelism hypothesis. The example is based on a data set reported by Finn (1974); the design has 4 groups, 2 dependent variables, and 3 covariates. Shown below are the results for the between-group factor (see Finn, 1974, p. C-54; see also Enslein, Ralston, & Wilf, 1977, p. 262), the summary for the covariates (see Finn, 1974, p. C49/50; Enslein et al., p. 258), and tests of the parallelism hypothesis (see Finn, 1974, p. C-45; Enslein et al., p. 255).

general manova	MAIN EFFECT: GROUP 1-GROUP	
Test	Value	p-level
Wilks' Lambda	.69357	
Rao R Form 1(6,80)	2.67672	.02031
Pillai-Bartlett Trace	.31790	
V(6,82)	2.58289	.02422

general manova	MULTIVARIATE TESTS Within Cells Regression 3 Covariates	
Test	Value	p-level
Wilks' Lambda	.90843	
Rao R Form 1 (6,80)	.65590	.68527
Pillai-Bartlett Trace	.09279	
V (6,82)	.66493	.67810

general manova	MULTIVARIATE TESTS OF PARALLELISM	
Test	Value	p-level
Wilks' Lambda	.74076	
Rao R Form 1 (18,62)	.55759	.91589
Pillai-Bartlett Trace	.27393	
V (18,64)	.56428	.91194

Example 27: Longley data set (linear regression) The so-called *Longley* data (Longley, 1967) is a well known data set for testing multiple regression programs for their ability to handle regression problems with redundant predictor variables. Shown below are the (partial) results computed by *STATISTICA*: Multiple Regression (see Longley, 1967; Elliott, Reisch, & Campbell, 1989, p. 296).

Dependent Variable: TOTAL
Multiple R: .997736942
Multiple R-Square: .995479005
Adjusted R-Square: .992465008
Number of cases: 16
 $F(6, 9) = 330.2853$ $p < .000000$
Standard Error of Estimate: 304.85407356
Intercept: -3482258.635 Std.Error: 890420.4

multiple regress.	Parameters	
variable	B	St. Err. of B
DEFLATOR	15.06187227143	84.914925774771
GNP	-.03581917929	.033491007772
UNEMPLOY	-2.02022980382	.488399681652
ARMFORCE	-1.03322686717	.214274163162
POPULATN	-.05110410565	.226073200069
TIME	1829.15146461400	455.478499142310

Note that there is a typographical error in the table presented in Elliott et al., 1989 (Table 4.3.1, p. 296); specifically, the *B* coefficient for *TIME* is 1829.151464614 (as reported in *STATISTICA*) and not 1829.15146416 (6 and 1 are reversed).

To our knowledge, *STATISTICA* is the only statistics program available on the market that will correctly compute and report regression coefficients for the Longley data set with this level of precision (Excel will correctly report the first 8 significant digits, Lotus will correctly report all 12 digits).

Example 28: Polynomial regression

Elliott, Reisch, and Campbell (1989, p. 295) present a data file to test polynomial regression. Shown below are the (partial) results computed by *STATISTICA*: Multiple Regression for the sixth degree polynomial fit (see Elliott, Reisch, and Campbell, 1989, p. 297). Note that this test is even more "demanding" than the previous one and an extremely low setting of the minimum tolerance parameter is required to obtain the parameter estimates.

Dependent Variable: Y HR
 Multiple R: .996793635
 Multiple R-Square: .993597550
 Adjusted R-Square: .990396325
 Number of cases: 19
 F (6, 12) = 310.3804 p < .000000
 Standard Error of Estimate: .308965061
 Intercept: 157.88215543 Std.Error: 73.68338

multiple regress.	Regression Weights		
	variable	B	St. Err. of B
X_KG		-330.97580114610	192.284963071360
P2		364.04271758509	201.286163909620
P3		-199.36108558038	108.400947552150
P4		58.11303781881	31.758798390784
P5		-8.60698967739	4.813032615799
P6		.50963834084	.295596359040

Example 29: Kaplan-Meier product limit estimates

Lee (1992, p. 25) discusses a data set first presented by King et. al. (1979). Shown below is part of the product-limit analysis for the *low-fat* group of rats as computed by *STATISTICA*: Survival Analysis (see also Lee, 1992, p. 74-75).

Kaplan-Meier (Product-limit) analysis			
survival	Note: Censored cases are marked with		
Case Number	Time	Cumulatv Survival	Standard Error
3	50.0000	.966667	.032773
12	56.0000	.933333	.045542
4	65.0000	.900000	.054772
13	66.0000	.866667	.062063
14	73.0000	.833333	.068041
9	77.0000	.800000	.073030
10	84.0000	.766667	.077220
5	86.0000	.733333	.080737
11	87.0000	.700000	.083666
...

Example 30: Comparing multiple samples of censored survival times

Lee (1992, p. 127) presents a data set of initial remission times for leukemia patients as a function of three treatments. Shown below is the summary of the comparison computed by *STATISTICA*: Survival Analysis (see also Lee, p. 127).

Variable: TIME
 Variable with censoring indicator: CENSORED
 Grouping variable: GROUP (3 Groups)
 Total number of valid observations: 66
 uncensored: 52 (78.79%)
 censored: 14 (21.21%)
 Chi-square = 3.61183 df = 2 p = .16434

Example 31: Proportional hazard regression for censored data

Crowley and Hu (1977) present an analysis of the well-known Stanford heart transplant data. Shown below are the (partial) results of the (Cox) proportional hazard regression analysis computed by *STATISTICA*: Survival Analysis (see also Brown, Engelman, Jennrich, 1990, p. 773).

Regression Results: Proportional hazard (Cox) regression
 Total number of valid observations: 65
 uncensored: 29 (44.62%)
 censored: 36 (55.38%)

Parameter Estimates				
Log-Likelihood of final solution: -87.867				
Variable	Beta	Standard Error	t-level	exponent Beta
AGE	.10909	.03329	3.27658	1.11526
ANTIGEN	-.04878	.47165	-.10342	.95239
MISMATCH	1.06372	.39460	2.69570	2.89713

Example 32: Exponential regression model for censored data

Lawless (1982, p. 287) discusses an example censored data set pertaining to lung cancer survival and fits to it an exponential regression model with six covariates (plus a constant). Shown below are the parameter estimates and their asymptotic standard errors computed by *STATISTICA*: Survival Analysis (see also Lawless, p. 288).

Parameter Estimates			
Variable	Beta	Std. Err	t-level
X1	.05442	.01082	5.0302
X2	.00887	.01977	.4484
X3	.00336	.01166	.2882
X4	.33865	.44556	.7601
X5	-.12069	.48623	-.2482
X6	-.86560	.58663	-1.4756
X7	-.28398	.38902	-.7300
Constant	4.74008	.40562	11.6861

Example 33: Stepwise discriminant function analysis and canonical analysis

The "classic" *Iris* data set (Fisher, 1936) is widely referenced to discuss discriminant function analysis. Shown below is the summary of the stepwise discriminant function analysis for those data, and the summary of the canonical analysis with all variables in the model (see also Jennrich 1977, pp. 92-94; Brown et al., 1990, p. 341-342).

Number of variables in the model: 4
 Wilks' Lambda: .023439
 Approx. F (8,288) = 199.145 p <0.00000

Summary of Stepwise Analysis					
Variable Entered	No. of vars.in	Lambda	F-level	df 1	df 2
PETALLEN	1	.05863	1180.16	2	147
SEPALWID	2	.03688	307.11	4	292
PETALWID	3	.02498	257.50	6	290
SEPALLEN	4	.02344	199.15	8	288

Standardized Coefficients for Canonical Variables		
Variable	Root 1	Root 2
PETALLEN	-.94726	-.401038
SEPALWID	.52124	.735261
PETALWID	-.57516	.581040
SEPALLEN	.42695	.012408
Eigenval.	32.19193	.285391

Example 34: Log-linear model (a 5-way frequency table)

Bishop, Fienberg, & Holland (1978, p. 103) present a complex 5-way frequency table describing the three-year survival of cancer patients in different locations. Shown below are the tests of all models of full order (see also Brown et al., 1983, p. 180; note that $\delta=0.5$ was added to each cell in the frequency table).

Results of Fitting all K-Factor Interactions						
log-lin.	K-Factor	df	Max.Lik. Chi-squ.	p	Pearson Chi-sq.	p
	1	8	632.156	.0000	881.251	.0000
	2	23	134.425	.0000	141.228	.0000
	3	28	30.909	.3212	31.233	.3069
	4	12	9.012	.7019	8.928	.7091

Example 35: Experimental Design: A 2(7-4) fractional factorial design**

Box, Hunter, and Hunter (1978, p. 391) present an example data set for a 2-level fractional factorial design; specifically the design is a 2**(7-4) fractional factorial. Shown below are the effect estimates as computed by *STATISTICA*: Experimental Design (see also Box, Hunter, & Hunter, p. 392).

experim. design	2**(7-4) design of resolution R = III TIME; m = 66.50000 s = 13.84609	
Effect	Effect Estimate	Sums of Squares
1:SEAT	3.50000	24.50
2:DYNAMO	12.00000	288.00
3:HANDBRS	1.00000	2.00
4:GEAR	22.50000	1012.50
5:RAINCOAT	.50000	.50
6:BREAKFST	1.00000	2.00
7:TIRES	2.50000	12.50

Example 36: Experimental Design: A second-order central composite (response surface) design

Box, Hunter, and Hunter (1978, p. 519) present an example data set for a 2-factor second-order central composite (response surface) design with two blocks. Shown below are the parameter estimates computed by *STATISTICA*: Experimental Design (see Box et al., p. 520).

experim. design	Parameter Estimates; Variable: YIELD 2**(2-0) 2nd order central composite design m=83.88333 s=4.39293 Intercept=87.3750	
Effect	Paramet.	Std.Err. of Par.
C vs. S	-.85003	.506913
TIME	-1.38374	.620843
DEGREES	.36199	.620843
1**2	-2.14377	.694129
2**2	-3.09379	.694129
1 by 2	-4.87500	.878000

Example 37: Experimental Design: A Taguchi robust design experiment (L18, S/N: Smaller-the-Better)

Phadke (1989, p. 82-83) discusses in detail the analysis of a robust design experiment pertaining to the manufacture of silicon wafers. Shown below is the summary ANOVA table computed by *STATISTICA*: Experimental Design for the *Surface Defect* data (a smaller-the-better problem; see also Phadke, p. 88, Table 4.6); note that as described in Phadke (p. 88), factor *Cleaning* was pooled into the error term.

experim. design	Analysis of Variance m = -45.362 s = 24.4841 * - effect pooled into error			
Effect	SS	df	MS	F
{1}TEMPERAT	4427.24	2	2213.62	27.26
{2}PRESSURE	3415.55	2	1707.77	21.03
{3}NITROGEN	1029.52	2	514.76	6.34
{4}SILANE	371.93	2	185.97	2.29
{5}SETT_TIM	378.28	2	189.14	2.33
*CLEANING	163.52	2		
Residual	568.46	7	81.21	

Examples 38-52: Analysis of Benchmark Data Sets

A standard set of benchmark data sets for the most common analyses was originally proposed by Elliott, Reisch, & Campbell (1989) and has since then been used in published reviews of statistical packages. Shown below are the results for all proposed benchmark analyses (and extensions of some of those tests designed to make them more demanding) as computed by *STATISTICA*.

Example 38: Descriptive statistics with small relative variances

Here are the results computed for the example data set proposed by Elliott et al. (p. 290). To demonstrate the precision of *STATISTICA* we have extended the test to extremely small relative variances (100000000001 to 100000000009).

Descriptive Statistics			
basic stats	N. of Cases = 9 (MD pairwise deleted)		
	Mean	St. Err.	ST. Dev.
V1	1005.0000	.91287092917528	2.73861266370720
V2	10005.0000	.91287092917528	2.73861266370720
V3	100005.0000	.91287092917528	2.73861266370720
V4	1000005.0000	.91287092917528	2.73861266370720
V5	10000005.0000	.91287092917528	2.73861266370720
V6	100000005.0000	.91287092917528	2.73861266370720
V7	1000000005.0000	.91287092917528	2.73861266370720
V8	10000000005.0000	.91287092917528	2.73861266370720
V9	100000000005.000	.91287092917528	2.73861266370720

Example 39: Independent group t-test

Here are the results computed for the t-test benchmark data set proposed by Elliott et al. (p. 290).

T-test; indep.var: FERTLZR		
basic stats	[1 gr.= PRESENT] [2 gr.= NEWER]	
	N. of Cases = 18	
	t	2-tailed p
Height	-2.988440	.008686

Example 40: Paired t-test

Here are the results computed for the paired t-test benchmark data set proposed by Elliott et al. (p. 290).

basic stats	Single t-Tests				
Comparison	t	p	N	E(X-Y)	D(X-Y)
HINDLEG-FORELEG	3.41379	.00770	10	3.3000	3.0569

Example 41: One-way ANOVA (test 1)

Here are the results of the one-way ANOVA benchmark (Example 1) proposed by Elliott et al. (p. 291).

MAIN EFFECT: FEED					
manova	1-FEED				
Univar. Test	Sum of Squares	df	Mean Square	F	p
Effect	4226.348	3	1408.783	164.64	.0000
Error	28.350	15	8.557		

Example 42: One-way ANOVA (test 2)

Here are the results of the one-way ANOVA benchmark (Example 2) proposed by Elliott et al. (p. 291).

manova MAIN EFFECT: CONDITN 1-CONDITN					
Univar. Test	Sum of Squares	df	Mean Square	F	p
Effect	10.6622	4	2.66556	3.3476	.0611
Error	7.1663	9	.79626		

Example 43: One-way repeated measures ANOVA

Here are the results for the one-way repeated measures ANOVA benchmark proposed by Elliott et al. (p. 292).

manova MAIN EFFECT: Drug 1-Drug					
Univar. Test	Sum of Squares	df	Mean Square	F	p
Effect	698.200	3	232.733	24.759	.0000
Error	112.800	12	9.400		

Example 44: Two-way ANOVA (balanced)

Here are the results for the two-way balanced ANOVA benchmark proposed by Elliott et al. (p. 292).

manova Summary of all Effects; design: 1-GENDER, 2-HORMONE						
Effect	df Effect	MS Effect	df Error	MS Error	F	p
1	1	70.31	16	22.898	3.071	.0989
*2	1	1386.11	16	22.898	60.534	.0000
12	1	4.90	16	22.898	.214	.6449

Example 45: Two-way ANOVA (unbalanced)

Here are the results for the unbalanced ANOVA benchmark data proposed by Elliott et al. (p. 293). We show here only the results for the Type III analysis (as "recommended" by Elliott et al., Table 3.7.2); note that Type I and II analyses can also be performed with *STATISTICA*: ANOVA/MANOVA.

manova Summary of all Effects (Type III SS) Design: 1-DRUG, 2-DISEASE						
Effect	df Effect	MS Effect	df Error	MS Error	F	p
*1	3	999.157	46	110.453	9.046	.0001
2	2	207.937	46	110.453	1.883	.1637
12	6	117.878	46	110.453	1.067	.3958

Example 46: Simple linear regression

Here are the results (computed via *STATISTICA*: Multiple Regression) for the data proposed by Elliott et al. (p. 294; note that the result reported in Elliott as *r-square* is in fact the result for the simple Pearson correlation coefficient *r*).

R: .726305400
R-Square: .527519535
Intercept: 4.910512449 St.Er: 6.627462 t(11)=.7409 p<.47

Variable	B	t(11)	p
HANDGUNS	.0376114423094	3.5044808186082	.00493

Example 47: Multiple linear regression (Example 1)

Here are the results for the data proposed by Elliott et al. (p. 295; note that the result reported in Elliott as *r-square* is in fact the result for the multiple correlation coefficient *r*).

Multiple R: .922119692
 Multiple R-Square: .850304726
 Intercept: 2.085724401

multiple regress. Regression Weights		
Variable	B	St. Err. of B
X1	.0569873379910	2.6131042380235
X2	1.0500229564602	.3262103147516

Example 48: Multiple linear regression (Example 2)

The next multiple regression benchmark proposed by Elliott et al., 1989 (p. 295, Example 2) is based on the well-known *Longley* data set (with redundant predictor variables, Longley, 1967). The results of this test are reported in Example 27, above. As mentioned before (see our Example 27), there is a typographical error in the table presented in Elliott et al., 1989 (Table 4.3.1, p. 296). Specifically, the *B* coefficient for *TIME* is 1829.151464614 (as reported in *STATISTICA*) and not 1829.15146416 (6 and 1 are reversed).

To our knowledge, *STATISTICA* is the only statistics program available on the market that will correctly compute and report regression coefficients for the Longley data set with this level of precision (Excel will correctly report the first 8 significant digits, Lotus will correctly report all 12 digits).

Example 49: Multiple linear regression (Example 3)

Here again are the (partial) results for the polynomial regression problem reported in Elliott et al. (1989, p. 297). Note that this test is even more "demanding" than the previous one and an extremely low setting of the minimum tolerance parameter is required to obtain the parameter estimates for the sixth order polynomial.

Dependent Variable: Y HR
 Multiple R: .996793635
 Intercept: 157.88215543 Std.Error: 73.68338

multiple regress. Regression Weights		
variable	B	St. Err. of B
X_KG	-330.97580114610	192.284963071360
P2	364.04271758509	201.286163909620
P3	-199.36108558038	108.400947552150
P4	58.11303781881	31.758798390784
P5	-8.60698967739	4.813032615799
P6	.50963834084	.295596359040

Example 50: A 2 x 2 contingency table and Fisher exact test

Here are the results for the 2x2 contingency table presented in Elliott et al. (p. 295).

Chi-square (N = 29) = 4.89 p < .0271
 Phi-Square = .168521
 Fisher Exact Probability (one-tailed): .032884

Example 51: An R x C contingency table (Example 1)

Here are the (partial) results for the 2x4 contingency table presented in Elliott et al. (p. 298).

df p
 Maximum Likelihood Chi-square: 9.51215 3 .023216
 Pearson Chi-square: 8.98718 3 .029477

log=lin. analysis	Expected Freq.: GENDER by HAIR_COL				
GENDER	HAIR_COL BLACK	HAIR_COL BROWN	HAIR_COL BLONDE	HAIR_COL RED	TOTAL
MALE	29.00000	36.0000	26.66667	8.33333	100.0000
FEMALE	58.00000	72.0000	53.33333	16.66667	200.0000
Total	87.00000	108.0000	80.00000	25.00000	300.0000

Example 52: An R x C contingency table (Example 2)

Here are the (partial) results for the 2x4 contingency table presented in Elliott et al. (p. 298). Note that the expected frequency for group *Negative/Days_0* is incorrectly reported in Elliott et al. as 14628 (and thus the expected frequencies in the first column do not add up to the marginal frequency); the correct expected frequency for this cell is 14628.5.

df p
Maximum Likelihood Chi-square: 62.4336 3 .000000
Pearson Chi-square: 283.047 3 .000000

log=in. analysis	Expected Freq.: DAYS by GROUP				
crossprd DAYS	GROUP DAYS_0	GROUP DAYS 1_2	GROUP DAYS 3_5	GROUP DAYS 6_9	Total
NEGATIVE	14628.50	312.0932	147.5712	.83777	15144.00
POSITIVE	42.50	.9068	.428855	.16223	44.00
Total	14671.00	313.0000	148.0000	56.00000	15188.00